

Mathematics Specialist Units 3,4
Test 5 2019

Section 1 Calculator Free
Differential Equations, Implicit Differentiation, Related Rates

STUDENT'S NAME _____

DATE: Monday 19 August

TIME: 20 minutes

MARKS: 20

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

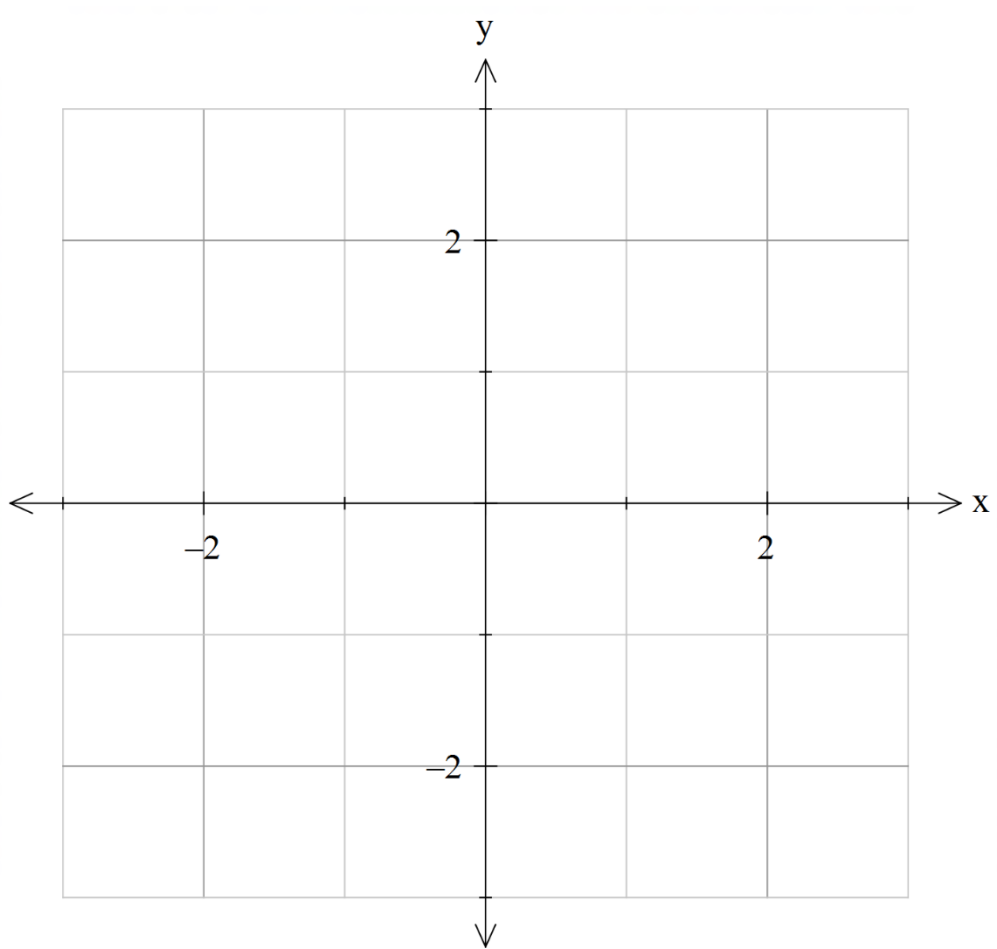
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Determine the general solution for the differential equation $\frac{dy}{dx} = x^2 y^2 - 2x^2 y + x^2$.

2. (5 marks)

- (a) On the axes below sketch the slope field for the differential equation $y' = y - x$. Use integer values of x and y only. [3]



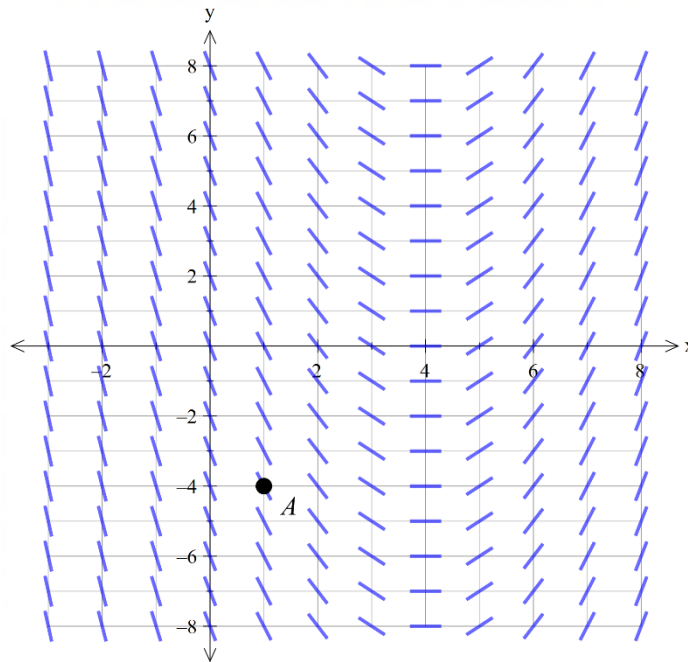
- (b) Sketch the particular solution to the differential equation on the axes above that passes through the point $(2,1)$. [2]

3. (4 marks)

Determine an expression for $\frac{dy}{dx}$ given $y^2 - ye^{\cos x} + 2x = \tan \frac{2\pi}{3}$

4. (7 marks)

A first order differential equation has a slope field as shown below.



- (a) Give a reason why the general solution of the differential equation for the slope field is quadratic and a reason why the differential equation is linear. [2]

The slope field at the point $A(1,-4)$ has a value of -3 .

- (b) Determine the equation of the curve containing point A . [5]



**Mathematics Specialist Units 3,4
Test 5 2019**

Section 1 Calculator Assumed
Differential Equations, Implicit Differentiation, Related Rates

STUDENT'S NAME _____

DATE: Monday 19 August

TIME: 30 minutes

MARKS: 30

INSTRUCTIONS:

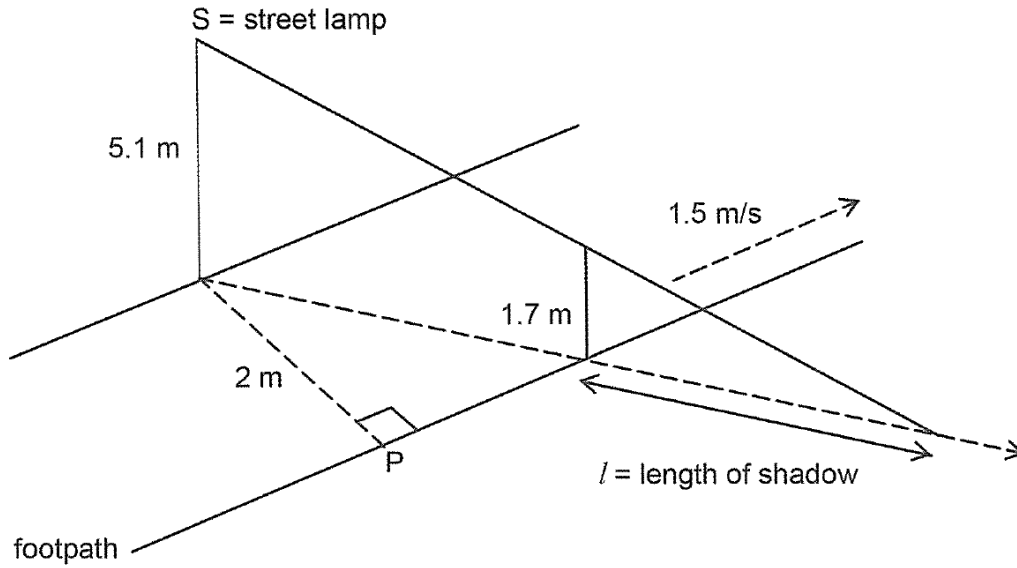
Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)



In the diagram above, P is the initial position of a boy, of height 1.7 metres, who is walking along a straight footpath in the direction shown.

S is the position of a street lamp of height 5.1 metres with its base 2 metres from P.

The street lamp will cast a moving shadow of the boy as he continues to walk along the footpath at 1.5 m/s.

- (a) If x metres is the distance walked by the boy from P, show that the length, l metres, of the boy's shadow is given by $l = \frac{1}{2}\sqrt{4+x^2}$. [3]

- (b) Determine the position of the boy when the rate of change of the length of the boy's shadow is 0.25 m/s. [3]

6. (9 marks)

The rate with which $\theta^\circ C$, the air temperature at altitude h metres, decreases with respect to altitude, is directly proportional to the sum between the air temperature and at altitude h and 273. The temperature at sea level, altitude = 0 metres, is assumed to be $20^\circ C$.

This relationship is given by $\frac{d\theta}{dh} = -k(\theta + 273)$ where k is a constant.

(a) Use calculus to determine an equation for θ in terms of h . [5]

Given $k = 3 \times 10^{-5}$;

(b) Calculate the air temperature at 3 km. [1]

(c) Determine the height at which the air temperature is $-60^\circ C$. [1]

(d) Determine the rate at which the temperature is changing when the height is 2 km. [2]

7. (7 marks)

A rotary sprinkler sprays a single jet of water out from its centre and rotates clockwise on its base at a speed of 4 revolutions per minute. The sprinkler's water jet reaches a maximum distance of 10 metres. The sprinkler is situated 5 metres away from the nearest point P on a straight wall.

(a) How fast is the jet of water moving along the wall when it is 80 cm from P. [4]

(b) What is the fastest speed the water jet reaches when moving across the wall and where does this occur? [3]

8. (8 marks)

With the removal of natural predators, the population of a species of marsupial on a nature reserve is expected to follow a logistic growth model given by $\frac{dP}{dt} = \frac{P}{4} - \frac{P^2}{2500}$ for t years.

(a) Determine an expression for P in terms of t in the form $P = \frac{K}{1 + Ce^{-at}}$ if there is an initial population of 80 animals. [4]

(b) How long will it take to reach half of the limiting population? [2]

(c) For what population is the growth rate quickest? [2]